Exam 4.

Due Wednesday April 27, 10 am.

1. Write the $L_{\omega_1\omega}$ sentence equivalent to "G is a torsion group".
2. Let c_0 be the set $\{x \in \mathbb{R}^\omega \colon \lim x = 0\}$ with coordinatewise addition and metric defined by $d(x, y) = \max\{|x(n) - y(n)| \colon n \in \omega\}$. Prove that c_0 is a Polish group. Prove that the equivalence relation E on $\mathbb{R}^\omega$ defined by $x \; E \; y$ if $x - y \in c_0$ is not Borel reducible to an S_∞-orbit equivalence relation.
3. Let G be a compact Polish group with zero-dimensional topology. Show that G is isomorphic to a closed subgroup of S_∞. *Hint.* Note that compact sets have only countably many clopen subsets.
4. Let X, d be the Cantor space with the minimum difference metric. Shopw that the group of all isometries of $\langle X, d\rangle$ is isomorphic to a closed subgroup of S_∞.