Exam 3.

Due Wednesday April 6 before class.

1. Let X be a Polish space, $E \subset F$ countable Borel equivalence relations on it such that E is Borel reducible to E_0 and every F-class consists of just finitely many E-classes. Show that F is Borel reducible to E_0.

2. Let X be a Polish space and E a countable Borel equivalence relation. Suppose that $B, C \subset X$ are Borel sets such that $X = B \cup C$ and $E \restriction B, E \restriction C$ are both Borel reducible to E_0. Show that the equivalence relation on the whole space X is Borel reducible to E_0.

3. Let G be a countable group. Show that the orbit equivalence relation of the shift action of G on $(2^\omega)^G$ is universal for all G-orbit equivalence relations.

4. Let G, H be countable groups, both amenable. Show that $G \times H$ is again amenable. *Hint.* Use the Reiter condition.