Exam 2.

Due March 24 before class.

Let X be a Polish space with a complete compatible metric d. Fill out the details in the proof that the group G of all isometries of $\langle X, d\rangle$ with pointwise convergence topology and the composition operation is a Polish group. Fix a countable dense set $D \subset X$.

1. Let $Y = X^D$ be the set of all functions from D to X. Show that the product topology is the same as the pointwise convergence topology on Y. This is to say, a sequence $\langle y_n : n \in \omega\rangle$ of elements of Y pointwise converges to $z \in Y$ if and only if it converges to z in the product topology.

2. Let $Z \subset Y$ be the set of all maps which preserve the metric d and have range dense in X. Show that Z is G_δ and so Z in the inherited topology is Polish.

3. Prove that if $\langle g_n : n \in \omega\rangle$ and k are elements of G then g_n converges pointwise to k if and only if $g_n \restriction D$ converges pointwise to $k \restriction D$.

4. Conclude that the map $h \colon G \to Z$ defined by $h(g) = g \restriction D$ is a homeomorphism of the spaces with the respective pointwise convergence topologies and therefore G is Polish.

5. Show that the composition and inverse operations on G are continuous.