Exam 3.

No cooperation. Due on Wednesday December 16 in my UF e-mail account or in my math department mailbox.

1. Let $B \subset \omega^\omega \times \omega^\omega$ be a Borel set. Show that there is a total continuous function $f \colon \omega^\omega \to \omega^\omega$ such that either $f \subset B$ or $f \subset (\omega^\omega \times \omega^\omega \setminus B)^{-1}$. Is this statement true with the interval $[0,1]$ replacing ω^ω? *Hint.* Let Players I and II play points $x, y \in \omega^\omega$ and let Player I win if $\langle x, y \rangle \in B$.

2. Let G be an open graph on a Polish space X. Show that either X is a union of countably many closed G-anticliques, or there is a perfect G-clique.

3. (Parametrized G_0 dichotomy.) Suppose that X, Y are Polish spaces and G is an analytic graph on $X \times Y$. Show that either $X \times Y$ can be covered by countably many Borel sets $B_n \subset X \times Y$ for $n \in \omega$ such that each vertical section of each B_n is a G-anticlique, or else there is a continuous homomorphism of G_0 to G whose image is contained in one vertical section of the space $X \times Y$.

4. Use the G_0 dichotomy to prove the Lusin separation theorem. (Note though that the Lusin separation theorem is used repeatedly in the proof of the G_0 dichotomy.) *Hint.* If $A, B \subset X$ are disjoint analytic sets then consider the graph connecting each element of A with each element of B. I think you also have to use the fact that analytic sets have the Baire property.