Exam 2.

Due Wednesday Nov. 4 before class.

1. Let X be a Polish space and E an analytic equivalence relation on it; i.e. $E \subset X \times X$ is analytic. (a) Show that if $A \subset X$ is an analytic set then the E-saturation of A, the set $\{x \in X : \exists y \in A \; x \; E \; y\}$ is analytic. (b) Use (a) to show that if $A_0, A_1 \subset X$ are disjoint analytic E-invariant sets, then there are disjoint E-invariant Borel sets $B_0, B_1 \subset X$ such that $A_0 \subset B_0$, $A_1 \subset B_1$.

2. Let $f\colon X \to Y$ be a Borel function between Polish spaces and $B \subset X$ be a Borel set such that $f \restriction B$ is injective. Prove that $f''B \subset Y$ is a Borel set.

3. Suppose that $B_0 \subset X_0, B_1 \subset X_1$ are Borel subsets of respective Polish spaces. Show that $B_0 \times B_1 \subset X_0 \times X_1$ is a Borel set.

4. Let $l_2 = \{x \in \mathbb{R}^\omega : \sum_n (x(n))^2 < \infty\}$. Show that $l_2 \subset \mathbb{R}^\omega$ is a $\mathbf{\Sigma}^0_2$-complete set.

5. Let $C = \{x \in [0,1]^\omega : x \text{ converges as a sequence}\}$. Show that $C \subset [0,1]^\omega$ is a $\mathbf{\Pi}^0_3$-complete set.